

Nonperturbative quark-gluon dynamics

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Abstract

We summarize recent results on the nonperturbative quark-gluon interaction in Landau gauge QCD. Our analytical analysis of the infrared behaviour of the quark-gluon vertex reveals infrared singularities, which lead to an infrared divergent running coupling and a linear rising quark-antiquark potential when chiral symmetry is broken. In the chirally symmetric case we find an infrared fixed point of the coupling and, correspondingly, a Coulomb potential. These findings provide a new link between dynamical chiral symmetry breaking and confinement.

1 Introduction

The relation between the two fundamental properties of QCD, confinement and dynamical chiral symmetry breaking (D χ SB), is surely a matter of utmost interest. Lattice calculations provide evidence that field configurations with nontrivial topological content may be at the heart of both phenomena [1, 2], but the fine details still remain elusive. Complementary to the strategy of identifying individual confining field configurations is the investigation of the correlation functions of the theory. Certainly, both confinement and D χ SB manifest themselves in strong, nonperturbative correlations at small momenta. In this talk we discuss these effects and present a novel link between confinement and D χ SB.

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2 Infrared behaviour of Yang-Mills theory

The infrared behaviour of Landau gauge Yang-Mills theory has been investigated in the past in a number of works in both the Dyson-Schwinger equations (DSE) framework [3–10] and also within the functional renormalisation group (FRG) [11–14]; for reviews see [15–17]. In the deep infrared, i.e. for external momentum scales $p^2 \ll \Lambda_{\text{QCD}}^2$, a general power law behaviour of one-particle irreducible Green functions with $2n$ external ghost legs and m external gluon legs has been derived [9, 10]:

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa+(1-n)(d/2-2)}. \quad (1)$$

Here, d is the space-time dimension. One can show that (1) is the only infrared solution in terms of power laws of both the complete hierarchy of DSEs and FRGs [13]. The anomalous dimension κ is known to be positive [4, 5] and is bounded by $\kappa \geq 0.5$ from below [5]. With the (well justified) approximation of a bare ghost-gluon vertex in the infrared one obtains $\kappa = (93 - \sqrt{1201})/98 \simeq 0.595$ [5, 6]. This value corresponds to an infrared vanishing gluon propagator and a strongly infrared enhanced ghost,

$$D_{\mu\nu}(k) = \frac{Z(k^2)}{k^2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right), \quad D_G(k) = -\frac{G(k^2)}{k^2}, \quad (2)$$

with dressing functions $Z(k^2) \sim (p^2)^{2\kappa}$ and $G(k^2) \sim (p^2)^{-\kappa}$. Such a behavior of the gluon propagator implies positivity violations and therefore may be interpreted as a signal for gluon confinement [3, 8].

An important consequence of (1) is the presence of a nontrivial infrared fixed point in the running couplings related to the primitively divergent vertex functions of Yang-Mills theory:

$$\begin{aligned} \alpha^{gh-gl}(p^2) &= \alpha_\mu G^2(p^2) Z(p^2) \sim \frac{\text{const}_{gh-gl}}{N_c}, \\ \alpha^{3g}(p^2) &= \alpha_\mu [\Gamma^{0,3}(p^2)]^2 Z^3(p^2) \sim \frac{\text{const}_{3g}}{N_c}, \\ \alpha^{4g}(p^2) &= \alpha_\mu \Gamma^{0,4}(p^2) Z^2(p^2) \sim \frac{\text{const}_{4g}}{N_c}, \end{aligned} \quad (3)$$

for $p^2 \rightarrow 0$. The infrared value of the coupling related to the ghost-gluon vertex can be computed [5, 7] and yields $\alpha^{gh-gl}(0) \simeq 2.972$ for $N_c = 3$.

3 Infrared behavior of quenched QCD

Based on the infrared solutions (1), one can also derive the analytical infrared behavior of the quark-gluon vertex [18]. To this end one has to carefully

distinguish the cases of broken or unbroken chiral symmetry. Whereas in the broken case the full quark-gluon vertex Γ_μ can consist of up to twelve linearly independent Dirac tensors, these reduce to a maximum of six when chiral symmetry is realized in the Wigner-Weyl mode. Correspondingly, a broken symmetry induces two tensor structures in the quark propagator, whereas only one is left when chiral symmetry is restored. In a similar way, chiral symmetry breaking reflects itself in every Green's function with quark content.

The presence or absence of the additional tensor structures turns out to be crucial for the infrared behavior of the quark-gluon vertex. When chiral symmetry is broken (either explicitly or dynamically with a valence quark mass $m > \Lambda_{\text{QCD}}$) one obtains a selfconsistent solution of the vertex-DSE which behaves like

$$\lambda^{D\chi SB} \sim (p^2)^{-1/2-\kappa}. \quad (4)$$

Here λ denotes generically any dressing of the twelve tensor structures. If, however, the Wigner-Weyl mode is realized one obtains the weaker singularity

$$\lambda^{\chi S} \sim (p^2)^{-\kappa}. \quad (5)$$

As a consequence the running coupling from the quark-gluon vertex either is infrared divergent ('infrared slavery') or develops a fixed point similar to the Yang-Mills couplings of eq.(3):

$$\alpha^{qg}(p^2) = \alpha_\mu [\lambda(p^2)]^2 [Z_f(p^2)]^2 Z(p^2) \sim \begin{cases} \frac{1}{p^2} \frac{\text{const}_{qg}^{D\chi SB}}{N_c} & : D\chi SB \\ \frac{\text{const}_{qg}^{\chi S}}{N_c} & : \chi S \end{cases} \quad (6)$$

(Here we use that the quark propagator is constant in the infrared, i.e. $Z_f(p^2) \sim \text{const}$ [19].) Note that in all couplings the irrational anomalous dimensions ($\sim \kappa$) of the individual dressing functions cancel in the RG-invariant products.

Finally, one can analyze the behavior of the quark four-point function $H(p^2)$ which includes the (static) quark potential. With (4) and (5), one obtains $H(p^2) \sim 1/p^4$ in the Nambu-Goldstone and $H(p^2) \sim 1/p^2$ in the Wigner-Weyl realization of chiral symmetry. This leads to a quark-antiquark potential of

$$V(\mathbf{r}) = \frac{1}{(2\pi)^3} \int d^3p \, e^{i\mathbf{p}\mathbf{r}} H(\mathbf{p}^2) \sim \begin{cases} |r| & : D\chi SB \\ \frac{1}{|r|} & : \chi S \end{cases} \quad (7)$$

which establishes the before mentioned link between dynamical chiral symmetry breaking and confinement.

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References

- [1] J. Greensite, Prog. Part. Nucl. Phys. **51** (2003) 1 [hep-lat/0301023].
- [2] C. Gattringer, Phys. Rev. Lett. **97**, 032003 (2006) [hep-lat/0605018];
- [3] L. von Smekal, A. Hauck and R. Alkofer, Annals Phys. **267** (1998) 1 [Erratum-ibid. **269**, 182 (1998)] [hep-ph/9707327].
- [4] P. Watson and R. Alkofer, PRL **86** (2001) 5239 [hep-ph/0102332].
- [5] C. Lerche and L. von Smekal, PRD **65** (2002) 125006 [hep-ph/0202194].
- [6] D. Zwanziger, Phys. Rev. D **67** (2003) 105001 [arXiv:hep-th/0206053]; Phys. Rev. D **69** (2004) 016002 [arXiv:hep-ph/0303028].
- [7] C. S. Fischer and R. Alkofer, PLB **536** (2002) 177 [hep-ph/0202202];
- [8] R. Alkofer, W. Detmold, C. S. Fischer and P. Maris, Phys. Rev. D **70** (2004) 014014 [hep-ph/0309077];
- [9] R. Alkofer, C. S. Fischer and F. J. Llanes-Estrada, Phys. Lett. B **611** (2005) 279 [arXiv:hep-th/0412330];
- [10] M. Huber, R. Alkofer, C. S. Fischer, K. Schwenzer, 0705.3809[hep-ph].
- [11] J. M. Pawłowski, D. F. Litim, S. Nedelko and L. von Smekal, Phys. Rev. Lett. **93**, 152002 (2004) [arXiv:hep-th/0312324].
- [12] C. S. Fischer and H. Gies, JHEP **0410**, 048 (2004) [hep-ph/0408089].
- [13] C. S. Fischer, J. M. Pawłowski, PRD **75** (2007) 025012 [hep-th/0609009].
- [14] J. Braun, H. Gies and J. M. Pawłowski, 0708.2413[hep-th].
- [15] R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281 [hep-ph/0007355].
- [16] C. S. Fischer, J. Phys. **G32** (2006) R253 [hep-ph/0605173].
- [17] R. Alkofer and J. Greensite, J. Phys. **G34** (2007) S3 [hep-ph/0610365].
- [18] R. Alkofer, C. S. Fischer and F. J. Llanes-Estrada, arXiv:hep-ph/0607293; C. S. Fischer, F. J. Llanes-Estrada, R. Alkofer and K. Schwenzer, in preparation.
- [19] C. S. Fischer and R. Alkofer, PRD **67** (2003) 094020 [hep-ph/0301094].